TEM Impedance and Cross Coupling for Small Circular Center Conductors in a Double Ridged Waveguide*

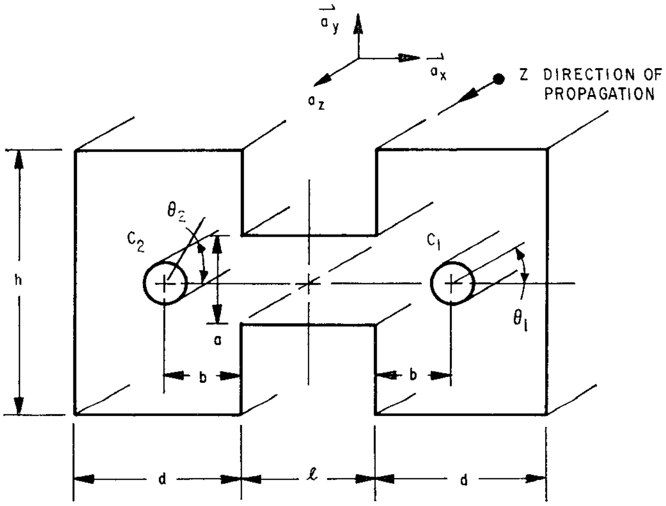
J. E. STORER†, SENIOR MEMBER, IRE, AND T. W. THOMPSON†, STUDENT MEMBER, IRE

Summary—The even and odd mode TEM impedances and cross-coupling coefficient were found for two small circular center conductors in a double ridge waveguide structure. Expressions were found by the use of a variational approximation for the case where the centers of the circular conductors lie on the horizontal center line of the guide; the conductors were placed symmetrically about the vertical plane of symmetry of the guide, and the conductors were placed a reasonable distance from the guide and from the region between the ridges. Results calculated from these expressions agree reasonably well with experimental data.

The experimental and theoretical results tend to indicate that proper placement of the two conductors in a double ridge guide could be used as a method of transmitting three different messages inside a single closed waveguide.

Introduction and Basic Equations

With the problems of conserving weight and space in today's aircraft and missiles, the use of one waveguide for several different transmission systems would be advantageous. Suppose we consider the waveguide system shown in Fig. 1. By launching TEM modes on conductors C_1 and C_2 and using the dominant waveguide mode, this system could transmit three different messages instead of the one message for the usual application of the ridged guide. For ideal conductors (conductivity $=\infty$) and no discontinuities in the guiding structures, there will be no coupling between the TE waveguide mode and the TEM modes. If the cross coupling between the TEM modes on C_1 and C_2 is small and the mode conversion due to discontinuities and imperfect conductors is small, this system would be a practical method of using one waveguide for three communication channels. This article will consider only the TEM even and odd mode impedances and the cross coupling between C_1 and C_2 for the case where the conductors C_1 and C_2 satisfy the requirements mentioned in the summary.



ρ = CENTER CONDUCTOR RADIUS

Fig. 1—Cross section of a waveguide system of two small circular center conductors inside a double ridge waveguide.

Basic Equations

The electric and magnetic field of the TEM modes of the system shown in Fig. 1 lie entirely in the transverse plane and are given by the following expressions:

$$\boldsymbol{E} = \text{Re}\left[-e^{j(kz-\omega t)}\nabla\phi(x, y)\right], \tag{1a}$$

$$\boldsymbol{H} = \frac{1}{\eta}(\boldsymbol{a}_z \times \boldsymbol{E}), \tag{1b}$$

where

$\boldsymbol{E}$ = electric field vector,

$\boldsymbol{H}$ = magnetic field vector,

$$\nabla = \boldsymbol{a}_x\frac{\partial}{\partial x} + \boldsymbol{a}_y\frac{\partial}{\partial y},$$

and

Re $e^{j(kz-\omega t)}$ = time and space variation of a wave travelling in the positive Z direction with a velocity ω/k.

The unit vectors in the x, y, and z directions will be noted by $\boldsymbol{a}_x$, $\boldsymbol{a}_y$, and $\boldsymbol{a}_z$, respectively. The quantity η is the characteristic impedance of uniform plane waves in the dielectric which fills the guide.

$$\eta = \sqrt{\frac{\mu}{\epsilon}}, \tag{1c}$$

where

μ = permeability of the dielectric filling the guide,
ϵ = dielectric constant of the dielectric filling the guide.

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† Sylvania Electronic Sys., Waltham, Mass.

Also, the potential function $\phi(x, y)$ satisfies the two-dimensional Laplacian equation,

$$\nabla^2\phi(x, y) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi(x, y) = 0. \tag{2}$$

The current in the center conductors will be given by i. It will be assumed that the current distribution varies angularly over the surface of the conductor. Thus,

$$i = \text{Re}\left[e^{j(kz-\omega t)}I(\theta)\right]a_z,$$

where θ is shown in Fig. 1.

The impedance of one wire to ground will be defined as

$$Z = \frac{\int_{\text{conductor}}^{\text{wall}} E\cdot ds}{\int_{\substack{\text{conductor}\\ \text{surface}}} i\cdot dA}, \tag{3}$$

where

$$ds = dx a_x + dy a_y,$$

$$dA = \rho d\theta a_z.$$

The coupling coefficient will be given by C and will be defined as

$$C = \frac{Z^e - Z^o}{Z^e - Z^o}\cdot \tag{4}$$

The superscripts e and o denote the even and odd modes, respectively. If no superscript appears, then the equation will be considered to be applicable to either mode (with suitable modifications). For the even mode, at any transverse plane, conductors C_1 and C_2 will be raised to the same potential with respect to the outer conductor. The current flow in C_1 and C_2 will be equal and in the same direction. For the odd mode, at any transverse plane, conductors C_1 and C_2 will be raised to opposite potentials. The resulting current flow in C_1 and C_2 will be equal in magnitude and opposite in direction. The coupling coefficient defined by (4) has been discussed in previous articles[1,2] for balanced transmission systems—thus the requirement that conductors be placed symmetrically about the vertical plane of symmetry.

Let us consider the region to the right of the vertical plane of symmetry as shown in Fig. 2. The origin of cartesian co-ordinate system will be taken as the center of the aperture. It is easily shown that potential function $\phi(x, y)$ must satisfy the following boundary conditions:

$$[\phi(x, y)]_{C_1} = \text{potential on the center conductor surface}$$

$$= V_0 \tag{5a}$$

$$[\phi(x, y)]_C = \text{potential on all conductor surfaces other than } C_1 = 0, \tag{5b}$$

$$\left[\frac{\partial\phi^e(x, y)}{\partial x}\right]_{x=-l/2} = 0, \tag{5c}$$

$$[\phi^o(x, y)]_{x=-l/2} = 0. \tag{5d}$$

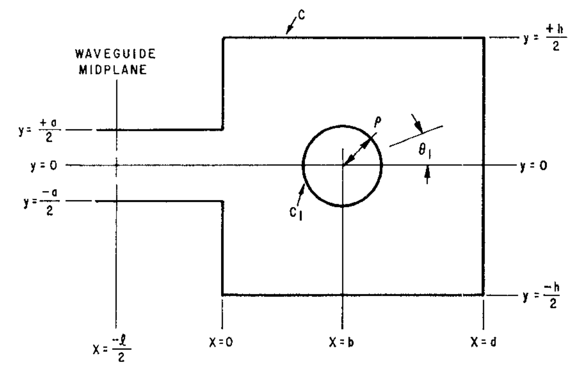

Fig. 2—Cartesian co-ordinate system in the transverse plane with the origin (0, 0) at center of the aperture.

Letting n_i denote a unit inwardly directed normal to the conductor surface and the subscript C_1 denote the value of the quantities at the surface of C_1, we can write the following boundary condition for the magnetic field at the surface of C_1:

$$i_{C_1} = -n_i \times H_{C_1}.$$

Using (1a) and (1b) and noting that the transverse current distribution is dependent only upon θ, it can easily be shown from the above that

$$I(\theta) = \frac{1}{\eta}\left[\frac{\partial}{\partial n_i}\phi(x, y)\right]_{x,y \text{ on } C_1}. \tag{5e}$$

It is readily apparent that this problem is not easily solved considering the differential equations. However, by considering an admittance Y defined as $1/Z$ and replacing the differential equations by integral equations and using a variational approximation, expressions for Z and C can be found. Thus, considering Y, we have

$$Y = \frac{1}{Z} = \frac{1}{V_0}\int_0^{2\pi} I(\theta)\rho d\theta$$

$$= \frac{1}{V_0\eta}\int_0^{2\pi}\left[\frac{\partial\phi(x, y)}{\partial n_i}\right]_{x,y \text{ on } C_1}\rho d\theta. \tag{6a}$$

The expression for the coupling coefficient now becomes

$$C = \frac{Y^o - Y^e}{Y^o + Y^e}\cdot \tag{6b}$$

[1] B. M. Oliver, "Directional electromagnetic couplers," Proc. IRE, vol. 42, pp. 1686–1692; November, 1954.

[2] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 75–81; April, 1956.

Formulation of the Integral Equations

Let us consider a region 1 to be the rectangular area enclosed by the lines

$$x = 0, \quad x = d, \quad y = \frac{+h}{2}, \quad y = \frac{-h}{2}.$$

For this region, a Green's function can be defined such that

$$\nabla^2 G_1(x, x_0, y, y_0) = -\delta(x - x_0, y - y_0), \tag{7a}$$

$$G_1 = 0 \text{ on all boundaries of Region 1.} \tag{7b}$$

The function G_1 can be thought of as the two-dimensional potential for a negative line charge located at x_0, y_0 inside a conductor having the same boundaries as Region 1. The delta function in (7a) has the property

$$\iint f(x, y)\delta(x - x_0, y - y_0)dxdy = \begin{cases} f(x_0, y_0) \text{ if the area integrated over includes} \\ \text{the point } x_0, y_0\text{; zero if the area integrated} \\ \text{over does not include the point } x_0, y_0. \end{cases}$$

A solution to (7a) and (7b) in terms of a Fourier series expansion in y is

$$G_1(x, x_0, y, y_0) = 2 \sum_{n=0}^{\infty} \frac{\cos\left(a_n \frac{y}{h}\right)\cos\left(a_n \frac{y_0}{h}\right)\sinh\left(a_n \frac{X^<}{h}\right)\sinh\left(a_n \frac{d - X^>}{h}\right)}{a_n \sinh\left(a_n \frac{d}{h}\right)}, \tag{7c}$$

where

$a_n = (2n+1)\pi$,
$X^<$ is the lesser of x, x_0,
$X^>$ is the greater of x, x_0.

Let Region 1′ be Region 1 excluding the center conductor cross section. If x_0, y_0 is supposed to lie in Region 1′ then

$$\phi_1(x_0, y_0) = \iint_{\text{Region } 1'} [G_1\nabla^2\phi - \phi\nabla^2 G_1]dxdy.$$

The above integral can be reduced from an area integral to line integral over the boundary by the use of Green's second identity. Thus

$$\iint_{\text{Region } 1'} [G_1\nabla^2\phi - \phi\nabla^2 G_1]dxdy = \int_{\text{boundary of Region } 1'} \left[G_1 \frac{\partial\phi}{\partial n} - \phi\frac{\partial G_1}{\partial n}\right] dl,$$

where the partial derivatives are with respect to an outwardly directed normal at the boundaries. The integrals above are reduced, noting the boundary conditions on ϕ and G_1 at the surface indicated. Thus,

$$\phi_1(x_0, y_0) = \int_{-a/2}^{a/2} \psi(y)\left[\frac{\partial G_1}{\partial x}\right]_{x=0} dy + \eta\int_0^{2\pi}(G_1)_{C_1} I(\theta)\rho d\theta - V_0\int_0^{2\pi}\left[\frac{\partial G_1}{\partial n}\right] C_1\rho d\theta,$$

where

$$\psi(y) = \phi(0, y) \quad \text{for } -a/2 \le y \le +a/2.$$

The last term in the expression above can be shown to be zero by the use of Green's second identity. Thus, for Region 1,

$$\phi_1(x_0, y_0) = \int_{-a/2}^{a/2} \psi(y)\left[\frac{\partial G_1}{\partial x}\right]_{x=0} dy + \eta\int_0^{2\pi}(G_1)_{x,y\to C_1} I(\theta)\rho d\theta. \tag{7d}$$

Turn now to a Region 2 which will be that area bounded by the lines $x = -l/2$, $x = 0$, $y = +a/2$, and $y = -a/2$. For this region we will define a second Green's function such that

$$\nabla^2 G_2(x, x_0, y, y_0) = -\delta(x - x_0, y - y_0)$$

$$G_2 = 0 \qquad \text{for } y = \pm a/2, \quad (-l/2 \le x \le 0)$$

$$\text{for } x = 0, \quad (-a/2 \le y \le +a/2)$$

$$\frac{\partial G_2^e}{\partial x} = G_2^o = 0 \quad \text{for } x = -l/2, \quad (-a/2 \le y \le +a/2),$$

where now x_0, y_0 is supposed to lie somewhere in Region 2. Writing G_2 as a Fourier series expansion

$$G_2^o = -2\sum_{n=0}^{\infty} \frac{\cos\left(a_n \frac{y}{a}\right)\cos\left(a_n \frac{y_0}{a}\right)\sinh\left[a_n \frac{\frac{l}{2} + X^<}{a}\right]\sinh\left(a_n \frac{X^>}{a}\right)}{a_n \sinh\left(a_n \frac{l}{2a}\right)}, \tag{8a}$$

$$G_2^e = -2\sum_{n=0}^{\infty} \frac{\cos\left(a_n \frac{y}{a}\right)\cos\left(a_n \frac{y_0}{a}\right)\cosh\left[a_n \frac{\frac{l}{2} + X^<}{a}\right]\sinh\left(a_n \frac{X^>}{a}\right)}{a_n \cosh\left(a_n \frac{l}{2a}\right)}, \tag{8b}$$

where $X^<$, and $X^>$ are those defined for G_1. By considering an analysis similar to that done for Region 1, it is easily shown that

$$\phi_2(x_0, y_0) = -\int_{-a/2}^{a/2} \psi(y)\left[\frac{\partial G_2}{\partial x}\right]_{x=0} dy. \tag{9}$$

By demanding that ϕ_1 reduce to V_0 on the conductor surface and that $\partial\phi_1/\partial x_0$ and $\partial\phi_2/\partial x_0$ be equal across the aperture ($x_0 = 0$),[3] we obtain the following integral equations

$$V_0 = \eta\int_0^{2\pi}\left[\lim_{x_0,y_0\to C_1} (G_1)_{C_1}\right] I(\theta)\rho d\theta + \int_{-a/2}^{a/2}\psi(y)\left[\frac{\partial G_1}{\partial x}\right]_{x=0} dy \tag{10a}$$

$$0 = \eta\int_0^{2\pi}\left[\frac{\partial}{\partial x_0}(G_1)_{C_1}\right]_{x_0=0} I(\theta)\rho d\theta + \int_{-a/2}^{a/2}\psi(y)\left[\lim_{x_0\to 0}\frac{\partial}{\partial x_0}\left(\frac{\partial G_1}{\partial x} + \frac{\partial G_2}{\partial x}\right)_{x=0}\right] dy. \tag{10b}$$

These equations are sufficiently complicated to exclude an explicit expression for $I(\theta)$. However, an expression for Y can be found which is stationary with respect to $\psi(y)$ and $I(\theta)$ and a variational principle can be applied using this expression.

Variational Approximation

Using (6a), (10a), and (10b) together with the symmetry properties of the Green's function, it can be shown that Y can be written as

$$Y = \frac{+2}{V_0}\int_0^{2\pi} I(\theta)\rho d\theta - \frac{\eta}{V_0^2}\int_0^{2\pi}\int_0^{2\pi}\left[(G_1)_{C_1}\right]_{x_0,y_0\to C_1} I(\theta)I(\theta_0)\rho^2 d\theta d\theta_0 - \frac{2}{V_0^2}\int_0^{2\pi}\int_{-a/2}^{a/2}\left[\frac{\partial}{\partial x_0}(G_1)_{C_1}\right]_{x_0=0} I(\theta)\psi(y_0)\rho d\theta dy_0 - \frac{1}{\eta V_0^2}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}\left[\frac{\partial}{\partial x_0}\left(\frac{\partial G_1}{\partial x} + \frac{\partial G_2}{\partial x}\right)_{x=0}\right]_{x_0=0} \cdot\psi(y)\psi(y_0) dy dy_0. \tag{11a}$$

This expression can be shown to be stationary with respect to independent variations of $I(\theta)$ and $\psi(y)$. To study an amplitude invariant form, let

$$I(\theta) = \frac{I_0}{2\pi\rho} K(\theta),$$

$$\psi(y) = \psi_0\Psi(y).$$

The expression for Y given by (11a) becomes

$$Y = \frac{2I_0}{V_0}\overline{R} - \frac{\eta I_0^2}{V_0^2}\Gamma_{11} - \frac{2I_0\psi_0}{V_0^2}\Gamma_{12} - \frac{\psi_0^2}{\eta V_0^2}\Gamma_{22}, \tag{11b}$$

where

$$\overline{R} = \frac{1}{2\pi}\int_0^{2\pi} K(\theta) d\theta,$$

$$\Gamma_{11} = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\left[(G_1)_{C_1}\right]_{x_0,y_0\to C_1} K(\theta)K(\theta_0) d\theta d\theta_0, \tag{11c}$$

$$\Gamma_{12} = \frac{1}{2\pi}\int_0^{2\pi}\int_{-a/2}^{a/2}\left[\frac{\partial}{\partial x_0}(G_1)_{C_1}\right]_{x_0=0} K(\theta)\Psi(y_0) d\theta dy_0, \tag{11d}$$

$$\Gamma_{22} = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2}\left[\frac{\partial}{\partial x_0}\left(\frac{\partial G_1}{\partial x} + \frac{\partial G_2}{\partial x}\right)_{x=0}\right]_{x_0=0} \Psi(y)\Psi(y_0) dy dy_0. \tag{11e}$$

[3] It should be noted that ϕ_1 and ϕ_2 given by (7d) and (9) satisfy the boundary conditions given by (5b), (5c) and (5d).

The values of I_0 and ψ_0 can be found by setting $\partial Y/\partial I_0$ and $\partial Y/\partial \psi_0$ equal to zero. The resulting values of I_0, ψ_0 and Y are

$$I_0 = \frac{V_0 \overline{R} \Gamma_{22}}{\eta(\Gamma_{11}\Gamma_{22} - \Gamma_{12}^2)}, \tag{12a}$$

$$\psi_0 = \frac{V_0 \overline{R} \Gamma_{12}}{(\Gamma_{12}^2 - \Gamma_{11}\Gamma_{22})}, \tag{12b}$$

$$Y = \frac{1}{Z} = \frac{(\overline{R})^2 \Gamma_{22}}{\eta(\Gamma_{11}\Gamma_{22} - \Gamma_{12}^2)}. \tag{12c}$$

For most accurate results, the potential and current distributions should be studied experimentally and some analytic function picked to represent these distributions. However, in other problems of this sort, a uniform current distribution is usually chosen and good results obtained. This then will be our choice. We cannot say much about the potential distribution except

1) $\psi(y) = 0$ at $y = \pm a/2$,
2) $\psi(y)$ is symmetrical about $y = 0$,
3) $\psi(y)$ is maximum at $y = 0$.

Note that conditions 2) and 3) presuppose that the circular conductors lie midway between the top and bottom of the ridge guide. A potential function satisfying the above conditions is the triangular form,

$$\Psi(y) = 1 - \frac{2}{a}\,|y|.$$

This form is easy to handle mathematically and yielded good results. Thus the trial functions should suffice as long as the following conditions are met:

1) The aperture is not too large,
2) The distance from every point on the boundary of Region 1 to the conductor is at least equal to the conductor radius, and
3) The aperture is not a dominant influence on the current distribution.

Using the above-mentioned trial functions, the expressions for $\overline{R}$, Γ_{11}, Γ_{12}, Γ_{22}, become $\overline{R} = 1$.

$$\Gamma_{11} = \frac{-1}{2\pi} \ln\left(\frac{\pi\rho}{2h}\right) + \sum_{n=0}^{\infty} \frac{e^{-a(d/nh)} - \cosh\left(a_n \frac{d - 2b}{h}\right)}{a_n \sinh\left(a_n \frac{d}{h}\right)}, \tag{13a}$$

$$\Gamma_{12} = 16\left(\frac{h}{a}\right) \sum_{n=0}^{\infty} \frac{\sinh\left(a_n \frac{d - b}{h}\right) \sin^2\left(a_n \frac{a}{4h}\right)}{a_n^2 \sinh\left(a_n \frac{d}{h}\right)}, \tag{13b}$$

$$\Gamma_{22} = -128 \sum_{n=0}^{\infty} \frac{\frac{1}{4}\left[\tanh\left(a_n \frac{l}{2a}\right)\right]^{\Gamma} + \left(\frac{h^2}{a^2}\right) \coth\left(a_n \frac{d}{h}\right) \sin^4\left(a_n \frac{a}{4h}\right)}{a_n^3}, \tag{13c}$$

where $a_n = (2n+1)\pi$.

For the even mode, $\Gamma = \Gamma^e = +1$.
For the odd mode, $\Gamma = \Gamma^o = -1$.

The above series converges quickly. The evaluation of Γ_{11} is shown in the Appendix. The evaluation of Γ_{11} and Γ_{12} was simplified by the fact that G_1 is essentially a potential function and its average over conductor surface is its value at center of the conductor ($x = b$, $y = 0$).

From the expressions for Y^e and Y^o, direct computation of the coupling coefficient from (6b) would be difficult and the values found probably erroneous. However, an expression for the difference ($Y^o - Y^e$) can be found and the coupling coefficient found using the second form of (6a).

Evaluation of Mode Coupling

For the waveguide system shown in Fig. 1 to be used as a three-channel communication system, we would use the dominant waveguide mode and the TEM modes associated with conductors C_1 and C_2. The waveguide mode will be operated in a frequency range such that all waveguide modes except the dominant mode are below cutoff (if the center conductors are small and located away from the region between the ridges, the change in cutoff frequency of the dominant waveguide mode is less than 10 per cent[4]). Under ideal conditions, the TE and TEM are considered orthogonal[5] and thus they will propagate independently. For an actual system, there

[4] Both experimental and theoretical results carried out by the authors and James D. Kellett under Contract No. AF 19(604)-5474, AF Cambridge Res. Ctr., Air Res. and Dev. Command, Bedford, Mass.

[5] See for example, N. Maruuvitz, "Waveguide Handbook," in "M.I.T. Radiation Laboratory Series," McGraw-Hill Book Co., Inc., New York, N. Y., vol. 10; 1951. See especially Sec. 1.2.

will be discontinuities and asymmetries resulting in mode conversion. This conversion is difficult to handle mathematically and this paper will be concerned only with the coupling between the TEM modes for ideal conductors as given by (4) and (6b). Thus, the coupling considered in this section is only that between the TEM modes.

Let us consider the following integral

$$\iint_{\text{region } 1'+\text{region } 2} [\phi^e \nabla^2 \phi^o - \phi^o \nabla^2 \phi^e] dxdy.$$

It is readily apparent that the above integral is identically equal to zero. However, by the use of Green's second identity, the above integral reduces to

$$0 = \int_0^{2\pi} \left[\phi^e \frac{\partial \phi^o}{\partial n_i} - \phi^o \frac{\partial \phi^e}{\partial n_i} \right]_{C_1} \rho d\theta + \int_{-a/2}^{a/2} \left[-\phi^e \frac{\partial \phi^o}{\partial x} + \phi^o \frac{\partial \phi^e}{\partial x} \right]_{x=-l/2} dy.$$

By the use of (5a), (5c), and (6a) it is easily shown that

$$Y^o - Y^e = \frac{1}{\eta V_0^e V_0^o} \int_{-a/2}^{a/2} \left[\phi^e \frac{\partial \phi^o}{\partial x} \right]_{x=-l/2} dy.$$

Substituting the form of ϕ given by (9), the previous equation yields

$$Y^o - Y^e = \frac{4}{\eta a^2} \sum_{n=0}^{\infty} \left\{ \left[\frac{a_n}{\sinh \left(a_n \frac{1}{a} \right)} \right] \times \left[\int_{-a/2}^{a/2} \frac{\psi^e(y)}{V_0^e} \cos \left(a_n \frac{y}{a} \right) dy \right] \times \left[\int_{-a/2}^{a/2} \frac{\psi^o(y)}{V_0^o} \cos \left(a_n \frac{y}{a} \right) dy \right] \right\}.$$

The above series converges very quickly so that only the first term need be considered. Thus,

$$Y^o - Y^e \approx \frac{64}{\eta \pi^3 \sinh \left(\frac{\pi 1}{a} \right)} \left[\frac{\Gamma_{12}}{\Gamma_{11}\Gamma_{22} - \Gamma_{12}^2} \right]^e \times \left[\frac{\Gamma_{12}}{\Gamma_{11}\Gamma_{22} - \Gamma_{12}} \right]^0. \tag{14}$$

Since the quantity $(Y^o - Y^e)$ will be a very small quantity, the sum $(Y^o + Y^e)$ can be replaced by some average value $2\overline{Y}$. This is calculated using (12c) and replacing Γ_{22} by $\overline{\Gamma}_{22}$ where

$$\overline{\Gamma}_{22} = -128 \sum_{m=0}^{\infty} \frac{4 + \left(\frac{h^2}{a^2} \right) \coth \left(a_n \frac{d}{h} \right) \sin^4 \left(\frac{a_n a}{4h} \right)}{a_n^3}.$$

Replacing Γ_{22}^e and Γ_{22}^o by $\overline{\Gamma}_{22}$ in (14), the following expression for C is obtained:

$$C = \frac{Y^o - Y^e}{Y^o + Y^e} = \left[\frac{\frac{32}{\pi^3}}{\sinh \left(\frac{\pi 1}{a} \right)} \right] \left[\frac{\Gamma_{12}^2}{\overline{\Gamma}_{22}(\Gamma_{11}\overline{\Gamma}_{22} - \Gamma_{12}^2)} \right]. \tag{15}$$

Results

Calculations for a ridged guide having the dimensions given in Table I were made. The characteristic impedance of either wire is given by $Z_C = \sqrt{Z^o Z^e}$.[1,2] Since Z^o and Z^e are very near, Z_C was calculated from (12c), replacing Γ_{22} by $\overline{\Gamma}_{22}$. For most dielectrics, the permeability is near that of space, and the dielectric constant is given by

$$\epsilon = \epsilon' \epsilon_0,$$

where

ϵ' = relative dielectric constant,

ϵ_0 = dielectric constant of free space.

Thus, η given by (1c) becomes $1/\sqrt{\epsilon'}\ 120\pi$ ohms. The quantity $Z_C\sqrt{\epsilon'}$ is given by

$$Z_C\sqrt{\epsilon'} = 120\pi \left(\Gamma_{11} - \frac{\Gamma_{12}^2}{\overline{\Gamma}_{22}} \right) \text{ ohms.} \tag{16}$$

The coupling coefficient was calculated using (15).

TABLE I

Dimensions of Type DR-19 Double Ridge Waveguide Manufactured by Technicraft Laboratories of Thomaston, Conn.

Dimension	Length in Inches
a	0.191
h	0.475
d	0.3845
l	0.256

Since we are considering the TEM mode, the impedance given by (3) will be proportional to the dc resistance of a two-dimensional configuration having dimensions proportional to those of the actual ridged guide.[6] This is to be expected since the potential for this two-dimensional configuration satisfies the same Laplacian equation and same boundary conditions as $\phi(x, y)$. The even and odd mode impedances were found and their average taken as $Z_C\sqrt{\epsilon'}$. These values are shown in Fig. 3 along with the values of $Z_C\sqrt{\epsilon'}$ calculated from (16).

[6] J. D. Kraus, "Electromagnetic," McGraw-Hill Book Co., Inc., New York, N. Y.; 1953. See especially Sec. 11.5.

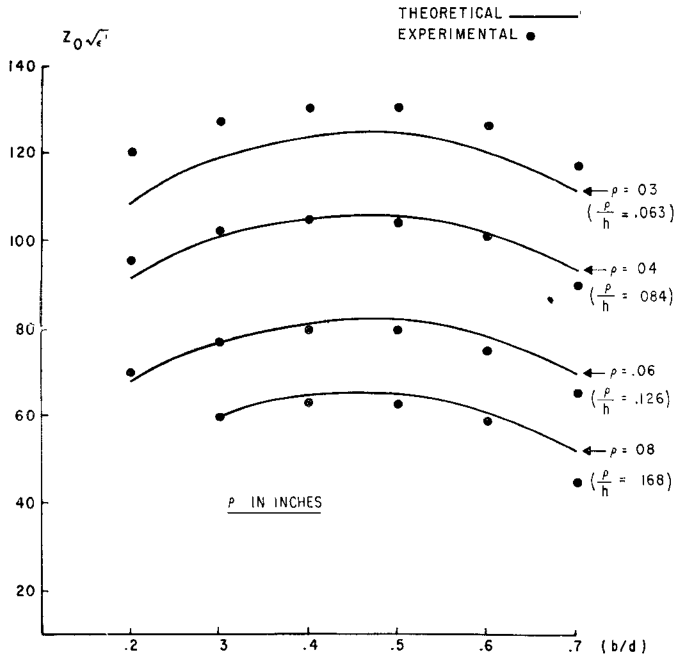

Fig. 3—Experimental and theoretical results for a double ridge guide having the dimensions given in Table 1.

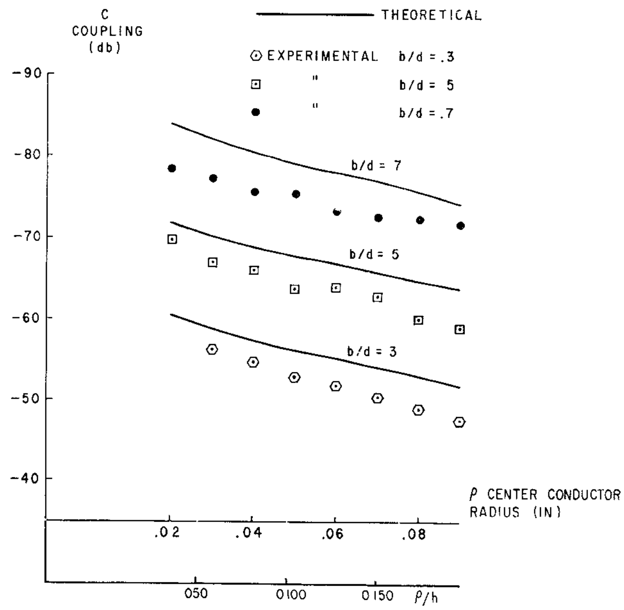

Fig. 5—Experimental and theoretical results for a double ridge guide having the dimensions given in Table I.

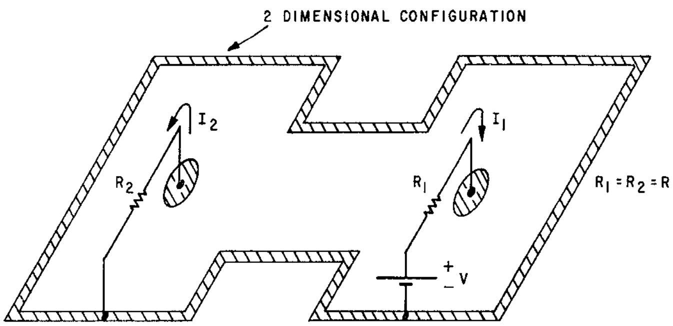

Fig. 4—Two-dimensional configuration used in finding the experimental results. The cross-hatched regions denote a highly conducting region. The experimental results shown in Figs. 3 and 5 were found with an electrolytic tank set-up based on the above.

Also the coupling coefficient, given by (4), can be found experimentally in a similar manner. It can be shown that

$$C = \frac{Z^e - Z^o}{Z^e + Z^o} = \frac{I_2}{I_1 - \frac{R}{V}(I_1^2 - I_2^2)},$$

where I_1, I_2, and V are shown in Fig. 4 for the case when $R_1 = R_2 = R$. The coupling coefficient found experimentally by this method is shown in Fig. 5 along with the theoretical calculated using (15).

As can be seen from Fig. 3 and 5, there is reasonable agreement between the theoretical and experimental results. Both the theoretical and experimental results indicate that this sort of system could be used as system for transmitting three messages inside a single closed guide.

The characteristic impedances are reasonable and the coupling data indicates that for all practical purposes, the only cross talk between channels would result from mode conversion due to discontinuities.

Appendix

Evaluation of Γ_{11}

For the uniform current distribution, Γ_{11} given by (11c) becomes

$$\Gamma_{11} = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\left[(G_1)_{C_1}\right]_{x_0,y_0\to C_1} d\theta d\theta_0. \tag{17}$$

The average of G_1 over the surface C_1 is the value of G_1 at $(b, 0)$. Thus, Γ_{11} becomes

$$\Gamma_{11} = \frac{1}{2\pi}\int_0^{2\pi}\left[(G_1)_{x=b,y=0}\right]_{x_0,y_0\to C_1} d\theta_0. \tag{18}$$

The Green's function in the above integral can be considered as the sum of free space Green's functions for the charge at x_0, y_0 and all of the images of the charge. The free space Green's function for a charge located at x_0, y_0 is

$$G_{fs} = -\frac{1}{2\pi}\ln\left[(x - x_0)^2 + (y - y_0)^2\right]^{1/2}. \tag{19}$$

Thus, G_1 becomes

$$G_1(x, x_0, y, y_0) = -\frac{1}{2\pi}\ln\left[(x - x_0)^2 + (y - y_0)^2\right]^{1/2} + \sum_{\text{all images of } x_0,y_0} G_{fs}. \tag{20}$$

Substituting the above in (18),

$$\Gamma_{11} = \frac{1}{2\pi}\int_0^{2\pi} \frac{-1}{2\pi} \ln\left[\rho^2\cos^2\theta + \rho^2\sin^2\theta\right]^{1/2} d\theta + \frac{1}{2\pi}\int_0^{2\pi}\left[\sum_{\text{all images of } b,0} G_{fs}\right] d\theta. \tag{21}$$

The first term is readily available. The second term is evaluated by replacing the integral by the field at the center. Thus,

$$\Gamma_{11} = \frac{-1}{2\pi}\ln\rho + \sum_{\text{all images of } b,0} (G_{fs})_{\substack{x\to b\\ y\to 0}}. \tag{22}$$

The second term in the above is evaluated using (20) where $x=b+\epsilon$ and limit is taken as ϵ approaches zero. Therefore,

$$\Gamma_{11} = \frac{-1}{2\pi}\ln\rho + \lim_{\epsilon\to 0}\left[G_1(b+\epsilon, b, 0, 0) + \frac{1}{2\pi}\ln\epsilon\right].$$

G_1 is given by (7c). Thus,

$$\Gamma_{11} = \frac{-1}{2\pi}\ln\rho + \lim_{\epsilon\to 0}\left[2\sum_{n=0}^{\infty}\frac{\sinh\left(a_n\frac{b}{h}\right)\sinh\left(a_n\frac{d-b-\epsilon}{h}\right)}{a_n\sinh\left(a_n\frac{d}{h}\right)} + \frac{1}{2\pi}\ln\epsilon\right]$$

$$= \frac{-1}{2\pi}\ln\rho + \sum_{n=0}^{\infty}\frac{e^{-a_n(d/h)} - \cosh\left(a_n\frac{d-b}{h}\right)}{a_n\sinh\left(a_n\frac{d}{h}\right)}$$

$$+ \lim_{\epsilon\to 0}\left[\sum_{n=0}^{\infty}\frac{e^{-a_n(\epsilon/h)}}{a_n} + \frac{1}{2\pi}\ln\epsilon\right] = \frac{-1}{2\pi}\left[\ln\left(\frac{\pi\rho}{2h}\right)\right] + \sum_{n=0}^{\infty}\frac{e^{-a_n(d/h)} - \cosh\left(a_n\frac{d-2b}{h}\right)}{a_n\sinh\left(a_n\frac{d}{h}\right)}.$$